Title: The construct mathematical resilience

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The construct mathematical resilience
By Clare Lee and Sue Johnston-Wilder

Summary

The pragmatic construct “mathematical resilience” describes the positive attributes which enable learners to engage with, learn and use mathematics both at school and beyond. There are four aspects to mathematical resilience:

- having a growth mindset, such that learners believe their mathematical capabilities can be developed through dedication and hard work;
- knowing that mathematics can be of personal value, is of value in the world and that the learner is valued within the community of learners;
- knowing how to work at learning mathematics, and;
- knowing how to find appropriate support to stay in the ‘growth zone’.

Mathematical resilience has been constructed in order to enable learners of mathematics, and those engaged in helping them, to act differently and thus prevent or mitigate the establishment of negativity towards mathematics. It works in opposition to, and equips learners to address, the negative emotional states of avoidance, anxiety and helplessness that frequently accompany learning mathematics. Learners who have developed mathematical resilience are better positioned to continue the study of mathematics. Mathematical resilience enables a simple message to be conveyed that learners, teachers, public and policy makers will understand, which is fully grounded in research and maintains an inner complexity.

Key words

Mathematical resilience, positive stance, growth mindset, value, changing learning, mathematics anxiety, avoidance, helplessness.
Introduction

Current educational research has developed a number of constructs, such as mathematics avoidance, mathematics anxiety and learned helplessness, each of which focuses on negative aspects and consequences associated with learning mathematics. The literature surrounding these constructs offers ideas and suggestions for treatment once a problem has developed, but typically does not concentrate explicitly on ways in which mathematics can be learned that do not result in the growth of such negative constructs. We have found that a pragmatic argument can be made for delineation of a positive construct which would enable learners to develop a positive stance towards mathematics. We call this construct ‘mathematical resilience’ (Johnston-Wilder & Lee, 2010a).

Resilience exists in wider psychological literature as a concept to describe the phenomenon of how some young people avoid negative consequences and succeed despite significant adversity. As we developed and worked with the construct ‘mathematical resilience’ we found that four aspects were important and pragmatically worthwhile to focus on. We will explain the four aspects in detail below. Alongside this discussion, we will give some examples of resilient approaches and teaching strategies, together with some evidence for regarding the four aspects as important. Mathematical resilience shares many characteristics with such constructs as self-efficacy, optimism, motivation and confidence and we will go on to show how each of these relates to mathematical resilience.

The construct ‘mathematical resilience’ allows learners to manage and protect themselves from unhelpful emotions that may arise when mathematics becomes difficult to learn. Resilient learners know that whilst learning mathematics requires struggle, appropriate support can be found and positive emotions that come from success can be experienced. Teaching for mathematical resilience enables learners to use mathematics effectively, and to acquire new mathematical skills when needed, to empower their day to day lives and careers. Learners of any age can develop the resilience they need to approach mathematics safely. All learners can learn mathematics in ways that do not cause them to develop negative traits.
The Need for Mathematical Resilience

A worldwide need

The construct mathematical resilience originated with work carried out in the UK, where there is an established long tail of underachievement in mathematics and a prevalence of negativity, including anxiety, towards the study of mathematics (Roberts, 2002, Callan 2015). Negativity towards mathematics seems to be a world-wide, but not global, problem; most countries have negativity towards mathematics within their populations (OECD, 2013). The report noted that “mathematics anxiety increased slightly since 2003” (p.98). The OECD also measured a related construct, mathematical helplessness, which has an average prevalence of about 30% in participating countries (ibid., p.80). It further noted that in Jordan, Thailand, Tunisia, Brazil, Qatar and Argentina at least 45% of students are reported as feeling helpless when doing a mathematics problem. The OECD report stated that the inverse relationship between mathematics anxiety and mathematics performance was strong among most participating countries and economies. “On average across OECD countries, greater mathematics anxiety is associated with a 34-point lower score in mathematics – the equivalent of almost one year of school.” (ibid., p.18)

OECD (2013) showed that disadvantaged students can succeed despite adverse socio-economic status, that is, some students were found to be resilient as the term is traditionally used in education. Across OECD countries, 31% of students from disadvantaged backgrounds were found to be resilient in this way, achieving beyond expectations. Such students succeeded by being engaged, motivated and holding strong beliefs in themselves and their abilities. In many countries, students’ motivation, self-belief and dispositions towards learning mathematics were found to be positively associated both with mathematical performance, and relative performance compared to other students in the same school. Furthermore, most mathematics teachers in participating countries believed that the social and emotional development of their students is as important as the acquisition of mathematics skills. Overall, the findings reported by OECD (2013) are consistent with our claim about the importance of developing “mathematical resilience” in learners of all ages.
Negative Influences on the Learning of Mathematics
That negative attributes such as mathematics anxiety, avoidance, learned helplessness and exclusion exist is well-established (see for example: Ashcraft, 2002; Hembree, 1990; Hernandez-Martinez & Williams, 2013). Negative emotional stress is also said to be frequently engendered by many traditional approaches to teaching mathematics (see e.g. Nardi & Steward, 2003). Experience and research (e.g. Johnston-Wilder, Lee, Garton, Goodlad & Brindley, 2013) has shown us that many learners are resilient in diverse aspects of their lives but seem reluctant, or unable, to bring that resilience to the learning of mathematics. Learners have told us (Lee & Johnston-Wilder 2013), for example, that whilst they readily talked to friends about any problems they had with their work in English or geography, they felt discouraged from doing so when engaged in mathematics work. Thus a specific notion of “mathematical resilience” is necessary.

Mathematics anxiety and avoidance
Mathematics anxiety and mathematics avoidance were important in our thinking about mathematical resilience because, as has been shown, these phenomena are prevalent in many countries (Ashcraft, 2002; Ma, 1999; OECD, 2013). In those countries, there is also a widespread concern about the decline in students prepared to study mathematics or mathematically demanding subjects after compulsory age (in the UK see for example Roberts, 2002; Johnston-Wilder, Brindley, & Dent, 2014; and elsewhere see USA: National Academies, 2007; and Australia: Slattery & Perpitch, 2010). Evidence (e.g. Johnston-Wilder et al., 2014) suggests that these two phenomena are linked. The widespread nature of mathematics anxiety and avoidance is also implicated when students drop out of studying subjects which become unexpectedly mathematical (for example, engineering courses or courses that involve statistics, such as psychology) (Johnston-Wilder, Lee, Garton, & Brindley, 2014).

Anxiety and avoidance are acquired phenomena. They may be either directly acquired and rooted in previous first-hand experiences or gained second-hand through talking to others about experiences
in mathematics. In a society where mathematics is routinely talked about negatively, such as UK, it is common to take a stance of antipathy towards mathematics (Callan 2015).

Mindset
One pervasive influence which is implicated in many people’s negative stance towards mathematics is the prevalent cultural assumption of a fixed mindset (Dweck, 2000). Those with a fixed mindset consider a person’s intellectual abilities have a fixed limit or ceiling beyond which they cannot grow. The terminology associated with a “fixed mindset” (for example: low ability pupil, reaching your potential) is part of the language used widely in the field of education in, for example, the USA and UK, and this discourse is particularly prevalent in relation to mathematics (Callan 2015). Many students undertake tests to measure their “ability” in mathematics, which seems to indicate an assumption that students possess a mathematics capability which remains fixed over time and is not susceptible to good teaching. Confusingly at the same time, many students are told that if they try harder, and make the effort, they will increase their attainment. In most secondary schools in England, learners are placed into ability groups, according to the results of specific tests. Such sorting, or stratification, seems to frequently be a self-fulfilling prophesy (Wiliam & Bartholomew, 2004), offering some justification to those who advocate the use of such ideas. The pupils in lower “ability” groups often suffer from lower teacher expectation and tend to be given less challenging work; as a result, the attainment gap between students in different groups widens (Boaler, 1997). Hence the school system may reinforce the idea that some people can "do mathematics", and others cannot, which is the basis of the fixed mindset. Such ideas, once ingrained, can make learning mathematics as an adult very difficult. However, these ideas can be challenged (Yeager, & Dweck, 2012); we argue that this does not often happen explicitly enough.

The discourse of fixed mindset also reinforces a form of elitism (Nardi & Steward, 2003), or exclusiveness, that surrounds mathematics. The assumption of elitism, the idea that only a minority can "do mathematics", means that the helplessness often seen in students of mathematics is accepted by the majority.
Learned helplessness  
Another prevalent phenomenon mentioned by the OECD (2013) is learned helplessness. The helpless person has learned that in a particular situation, certain responses and outcomes are independent of their own effort (Abramson, Seligman, & Teasdale, 1978). For example, a student may try to remember all the steps in a mathematical procedure, inadvertently miss one out and arrive at the wrong answer. If this happens frequently over time, the student learns that their effort does not achieve the outcome of the correct answer. Thus they conclude that their effort is not enough to achieve success in mathematics and next time the learner expends less effort.

The constituent ideas of fixed mindset, elitism, anxiety, exclusion and helplessness all result in a large proportion of society that wants nothing to do with mathematics and creates a society in which mathematics avoidance is implicitly condoned, and in which being willing and able to engage with mathematics gives significant additional cultural capital to the elite few.

Mathematical Resilience  
Instead of working in ways that unwittingly develop anxiety and avoidance, learned helplessness and exclusion, teachers and others involved with the promotion of mathematical learning, such as coaches and parents, can work explicitly to develop mathematical resilience. Rather than focus on the negative, in the spirit of positive psychology and medicine (e.g. Tennant, Hiller, Fishwick, Platt, Joseph, Weich, & Stewart-Brown, 2007), it seems to be more productive to focus on the positive: attitudes and beliefs that enable learners to approach mathematics positively and that future employers say they want to see. The construct ‘mathematical resilience’ is designed to work explicitly against negative influences. Mathematical resilience is above all characterised by a ‘can do’ approach to any new mathematics encountered, a willingness to put in effort to develop fluency, and an ability to assemble whatever support is needed to overcome any barriers to mathematical growth. Learned helplessness and mathematics anxiety can be overcome.
Why Mathematical Resilience?

Resilience is particularly needed in learning mathematics, due in part to the characteristics of that subject, the prevalent fixed mindset and “The overwhelming impression [...] that mathematics is a very difficult subject for most children” (Hart, 1980, p.209). Any survey of people in UK in general will quickly reveal that many not only “hate maths” but also have a story to tell about just why they feel such antipathy. A further and very important reason that mathematics is often shunned is the performativity agenda as identified by, among others, Hernandez-Martinez and Williams (2013).

“Our research does pose this important question, then: given the predominant educational culture of performativity to which schools and, increasingly, universities, are subjected, how can mathematics teaching allow for reflexivity and for the construction of relevant capital for the majority and, ultimately, for the creation of such resilience among all students?” (Hernandez-Martinez & Williams, 2013, p.57)

For many people, engaging in mathematics is a form of adversity, which they are required to confront until their school leaving age, but which they choose to avoid as soon as possible thereafter. Much, but certainly not all, teaching of mathematics leads to many learners viewing mathematics as being the rapid performance of technically correct steps with no obvious reason, steps which rely heavily on memory (Ward-Penny, Johnston-Wilder, & Lee, 2011). The creativity and puzzlement, the experimentation and mis-steps that characterise a working mathematician’s life (Burton, 2004) are rarely reflected in the learner’s experience. Fluency with procedure is often valued over understanding of that procedure or knowing how that procedure connects with other aspects of mathematics (see, for example, Ofsted, 2012). Working memory is reliant upon being calm (Ashcraft & Krause, 2007) and anxiety interferes with the recall that students consider to be required (Nardi & Steward, 2003). The way that mathematics is often presented causes many, but not all, learners to experience adversity, sometimes to the point of trauma. Stories of such adversity that we know of include a student reduced to tears because she could not remember a given procedure; the more she was instructed to “just think” the more impossible it became to think
clearly. Thirty years later, this person still recalls how she felt during and after the incident, and she has avoided mathematics ever since.

It would, however, be wrong to say that students should always learn and practice mathematics in a completely smooth and untroubled way. Indeed, one might argue that if a student is always experiencing mathematics without challenge, they will be ill-prepared for struggle later in their studies (Ward Penny, et al. 2011). There are barriers to overcome when studying mathematics and struggle is part of learning. Connections must be actively sought; fluency is necessary and only comes with familiarity. Until a learner can confidently conjure one procedure, often another aspect will remain out of reach (McGowen & Tall, 2010). Stigler and Hiebert (2009) have shown how pervasive path smoothing (Wigley, 1992) is and how some teachers of mathematics in U.S.A. have come to understand that their job is to remove barriers rather than to help learners in their struggle. Wigley (1992) emphasises that a teacher’s role is to help learners build the resilience to engage in the struggle to overcome the barriers that mathematics presents.

Growing Mathematical Resilience
The four aspects of mathematical resilience (growth mindset, personal value of mathematics, knowing that mathematics requires struggle and knowing how to recruit support in pursuing mathematical learning) chime with aspects postulated to be important by others, including Bandura (1995), Dweck (2000), Williams (2014), Hernandez-Martinez and Williams (2013) and Ryan and Deci (2000), but are explicitly focused on the learning of mathematics. Kookén, Welsh, Megan, McCoach, Johnston-Wilder, & Lee (2015) explored the measurement of the construct in people who could be considered to have mathematical resilience. They concluded that there were at least three affective dimensions to becoming sufficiently mathematically resilient to engage in university level study of subjects with a high mathematical content. The dimensions were:

- Value: the experience that mathematics is a valuable subject and is worth studying;
• Struggle: the recognition that struggle with mathematics is universal even for people who have a high level of mathematical skill;

• Growth: the belief that all people can develop mathematical skill and that everyone can learn more mathematics with effort and support.

Further work in this area (Johnston-Wilder & Lee, 2010b; Lee & Johnston- Wilder, 2014; Goodall & Johnston-Wilder, 2015) has led us to believe that, when considering how to enable people to develop mathematical resilience, there are four aspects that need to be considered which we will go on to discuss in depth:

1. Growth mindset: developing an incremental or growth mindset

2. Value: understanding and experiencing mathematics as important in society and also personally. This aspect is also about the value of the individual as part of the mathematical community.

3. An understanding of how to work at mathematics: that progress in mathematics requires struggle, curiosity and perseverance as well as learning to manage the emotions that come with learning something new.

4. Knowing how to recruit support: an awareness of the value of collaboration, the use of conjoint agency to aid in the struggle to grow mathematical knowledge, skills and understanding.

1. A Growth Mindset

Pupils who have developed mathematical resilience are characterised by a growth mindset (Yeager & Dweck, 2012). Students who believe that intellectual abilities can be developed, as opposed to being fixed at a certain level, tend to show higher achievement (Yeager & Dweck, 2012) and to develop resilience. Encouraging students to develop a growth mindset is ultimately about reminding students continually that when they practice new or different ways of doing mathematics, their brains develop (‘just like a muscle’) and they become more adept at dealing with the challenges of learning mathematics. A growth mindset emphasises the malleability of the brain, including adult
brains. Focusing on the importance of adapting strategies, and learning skills that let learners use their brain in smarter ways, helps develop resilience when faced with novel and challenging problems. Developing a growth mindset needs an emphasis on getting better at something through effort, acknowledging that “you can’t do it yet” but given time, persistence and appropriate help (not too much) from others, “you will be able to”. The brain grows every time a learner makes connections and learns something new (Willis, 2007)).

2. **Value**

People who have developed mathematical resilience can be characterised by an understanding of the value of mathematics. It is seen as a way of thinking about topics and issues that interest them and a way of modelling the world, as well as a way of accruing personal cultural capital. Resilient learning demands an understanding of the purpose of the skills being learned in terms of personal, employment and cultural objectives (sometimes historical, sometimes current). Furthermore resilient learners feel included in the community of those learning mathematics. They understand that their own position as part of the community of mathematicians or people who use mathematics is of value. Those who are studying or are going to study mathematics or mathematically demanding subjects will quickly find that their mathematical qualifications are valued in the jobs market.

It may seem self-evident that mathematical learning is of value but this is not always the case and learners often feel excluded. Brown, Brown, & Bibby (2008) state that amongst several factors that have an impact on the uptake of mathematics at A-level are that mathematics is perceived to be ‘hard’, ‘boring’ and ‘useless’. For most financial transactions, the use of a calculator is sufficient, and cash registers now work out any change needed so even basic arithmetic may be perceived as not in demand in employment. The mathematics that will empower winning the best deal, building a stable structure, navigating a small boat or understanding whether a chart is a fair representation of the facts, frequently hides against its background and requires a certain amount of exploration to uncover. Mathematics is hard for some people to value when it cannot be seen in the valued aspects of their life. Sadly many mathematics teachers we have met are content to study mathematics for its
own sake, and they are not well-prepared for the question “when will we use this in real life?” Many teachers of mathematics have not recognised the connection between algebraic skills and choosing a mobile phone tariff or preparing a large meal.

Algebraic thinking can be made more inclusive; for example, Lugalia (2015) had girls running to algebra lessons. The software she used was designed with Bruner’s (1966) enactive-iconic-symbolic model as its foundation. This model is about accessibility; if learners are struggling with a symbolic algorithm for factorising a quadratic expression, they could be introduced to an iconic representation using diagrams, which may make the task more accessible. If learners are struggling with expanding brackets, such as \((x+2)(x+1)\), they may be included by using some pieces of card, squares to represent the \(x^2\) terms, strips 1 by \(x\) to represent the \(x\) terms and unit squares. If the learners are then invited to make the shapes into a rectangle and describe its dimensions, they will find themselves enacting the expansion. One teacher, who had taught for many years in two countries, introduced this approach, and was astounded at how capable all her pupils at age 11-12 years became with quadratic expansions, when most of her previous students had struggled with a purely symbolic approach throughout five years of secondary schooling. She recognised she had unwittingly excluded many students from algebraic thinking.

By paying attention to inclusion, all learners are invited to become valued members of a community, engaging in mathematical thinking and thus beginning to experience personal value in mathematics.

3. Knowing how to work at mathematics – the growth zone

Learning and using mathematics is not easy, everyone has barriers to overcome, mistakes are made and people get stuck (Mason, Burton, & Stacey, 2010); mathematics needs perseverance (Williams 2014, Burton, 2004). However, resilient learners know that they are not struggling alone. There is a community ready to offer support and appropriate help, from working alongside, to encouraging, to suggesting resources, to reminding of known but forgotten processes. Mathematics does not need enormous memory power, this is a common misconception (Ward-Penny et al., 2011); connections
can be explored, patterns seen and collaboration is very productive (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997, Swan, 2006).

Resilient learners exercise agency in their learning (Lee & Johnston-Wilder, 2013, Ryan and Deci 2000). Learners develop agency through being allowed choice, independence and the opportunity to experiment. A facilitator of learning mathematics in a resilient manner will emphasise that there is no one fixed way of doing things, but rather that each learner must start from where they are, using what they know already (McGowen & Tall, 2010) and work to fit the new learning into their existing schemata. Mistakes will be made, which is important as mistakes indicate an opportunity to learn (Mason, et al., 2010).

Developing mathematical resilience can be especially hard for those who have previously developed anxiety and learned helplessness. In helping adults and teenage learners to sufficiently overcome such anxieties and to begin to approach mathematical ideas with resilience, we developed a model we call “The Growth Zone Model”. In developing the model, our thinking was influenced by our study of Vygotsky (1978) but this model is purposely simple in its construction. It is designed to enable any learner to articulate and begin to understand the emotions they may be feeling as they approach mathematical learning.

Figure 1 The Growth Zone Model

The model consists of three zones: comfort, growth and danger.

**Comfort (green):** In the comfort (green) zone a learner will feel comfortable, when working at mathematics they will be doing something which they know about and can do on their own. Thus the green zone is useful for consolidating ideas, developing fluency and the knowledge that you understand and can use a particular skill in mathematics. However if learners stay too long in the green zone they will not learn as much as they could, and it is quite likely that they will suffer boredom and a lack of stimulation.
**Growth (amber):** In the growth (amber) zone the learner is being challenged to some degree. The more time the learner can stay in this zone the more they will learn and progress. In this zone, the challenge is high but not too high, there is excitement as novel ideas are being explored and there is the satisfaction of overcoming barriers and grasping at understanding. There is sufficient adrenalin to persevere when things seem tough. It is a positive, but somewhat exhausting, place to be, hence when fatigue sets in, learners may choose to resort to the green zone to consolidate the progress made. However there is always the risk that the challenge might get too high, the learners may not feel able to pause and draw breath when they need to and thus they may begin to feel unsafe and unable to safeguard themselves.

**Danger (red):** The danger (red) zone is marked not only by anxiety but also panic, and it is sometimes the kind of panic that triggers the “fight, freeze or flight” mechanism which means that thinking and reasoning are almost impossible; the pre-frontal cortex goes ‘off-line’. Many learners we have encountered have so many memories of being in the red zone when faced with mathematics that they find themselves in the danger zone when asked just to talk about doing mathematics.

We have found that this growth zone model enables even the most anxious learner to articulate what they are feeling and therefore to consider how they might overcome their emotional barriers sufficiently to spend just a short time in the growth zone. One insightful learner suggested to her teacher: “the trouble is that in maths my growth zone is too narrow”. Collaborative working with conjoint agency, where each member of the group is charged with supporting, as well as being open to receiving support, can enable more people to access and stay longer in their growth zone.

4. **Knowing how to gain support**
   Resilient learners of mathematics know that mathematics requires struggle but they also know how to access help and support in that struggle. Some learners may need to be shown successful models of mathematical thinking and reasoning, all need to be listened to because in formulating thinking to communicate ideas to others, that thinking becomes clarified (Lee, 2006). Furthermore when the
“sticking point” becomes obvious, a more capable peer will be able to question the reasoning or offer targeted and appropriate support. Collaborative discourse may enable students to learn to ask themselves the right questions (Lee, 2006).

In order to work collaboratively, learners need to build up sufficient language or vocabulary so that they can use enough of the mathematics register to be able to express and explore mathematical ideas (Lee, 2006). This is not to say that they must use the “right” words but rather that they see a need to express mathematical ideas themselves and thus seek a way to do so effectively. Effective mathematics communicators can explore their own understandings and connections and support others. Mathematical resilience is based within a social constructivist domain (Vygotsky, 1978); expressing mathematical ideas and talking about mathematical learning within a mathematical community are both vital aspects of developing the resilience that allows for learning mathematics.

Digital technologies have a role to play here. Search engines furnish a process or ready-made procedure which may yield solutions to a problem and software such as Grid Algebra or dynamic geometry programmes provide a safe place to express ideas, experiment, think and learn. The potential to make mistakes and to learn from those mistakes is high when using IT as feedback tends to be immediate and having ‘another go’ straightforward. The transitory nature of attempts on screen appeals to many people who suffer acute anxiety, while they build their resilience.

Relating Mathematical Resilience to other constructs.

Resilience
Resilience seemed an important term to use as it describes the attributes needed to overcome adversity, which it appeared to us many learners were experiencing when learning mathematics.

Along with several other authors (for example Knight, 2007; Hernandez-Martinez & Williams 2013) we think it is important to see resilience as a socio-cultural construct which communities can address, not an individual psychological construct.
Resilience has many subtly different definitions in literature. The term resilience grew out of the work of developmental psychologists, such as Rutter (1987) and Garmezy (1991), who recognized that, among groups of children believed to be at high risk of developing particular difficulties, many individuals remained undamaged by significant adversity. Others offer clarification, for example, “resilience refers to a dynamic process encompassing positive adaptation within the context of significant adversity.” (Luthar, Cicchetti, & Becker, 2000, p.543), or resilience is, “both the capacity to be bent without breaking and the capacity, once bent, to spring back” (Vaillant, 1993, p.248). Resilience is further characterized as: “…mechanisms and processes that lead some individuals to thrive despite adverse life circumstances.” (Galambos and Leadbeater 2000, p.291). Commonality in the definitions resides in the capability of some individuals to recover or to continue on a positive path when faced with significant adversity.

Literature surrounding those children who are resilient indicates that there are three sets of factors implicated in the development of resilience: (1) attributes of the children themselves, (2) aspects of their families, and (3) characteristics of their wider social environments, (e.g. Masten & Garmezy, 1985; Werner & Smith, 1992).

1. The resilient attributes of the learner can be influenced, either adversely or positively, by the persons who are supporting their learning. Ensuring that this influence is as positive as possible requires the learner and supporter to become aware and mindful of affective traits and emotions as part of developing resilience. This is a crucial part of mathematical resilience.

2. That the family has a significant effect on building resilience is also recognised within our work; parents can learn to enable their child’s learning of mathematics more effectively without learning any more mathematics (Goodall & Johnston-Wilder, 2015; also Goodall, Johnston-Wilder & Russell, this volume).
3. Resilience is affected by the psycho-social aspects of the environment in which the student is learning mathematics. Where such an environment is transformative (Mezirow, 2000), focusing on the development of interaction between learners, then resilience is likely to be an outcome for the learner. In a transformative environment, critical and constructive thinking methods enable learners to look deeply into practices, to develop creative ways of thinking, to improve problem-solving skills and to work collaboratively. “Thus, the learning of resilience fits well with the transformative educational framework.” (McAllister & McKinnon, 2009, p.375).

Growing resilience
We consider that “resilience is not a quality that some possess and others do not” (Knight, 2007, p.545) but rather that learners can grow their resilience when learning mathematics. All learners need some resilience to overcome the significant challenges that learning can at times present and “there is convincing evidence that individuals can learn or acquire resilient qualities” (McAllister & McKinnon, 2009, p.374). Schools, teachers and supporters of learning beyond schools can provide an environment that fosters resilience. For example, Borman and Overman (2004) suggest that a safe and orderly environment and positive teacher-student relationship are likely to promote resilience in poor ethnic minority students. Knight concludes that teachers need to have “a 'resiliency attitude' so that they see children and young people as competent, and focus on their strengths rather than deficits.” (Knight, 2007, p.544.) Exactly how the focus on “strengths rather than deficits” is done is important, because if anyone trusted offers comfort to a student that reinforces the idea that a weakness is inherited or to be expected, then that may further contribute to the notion of a fixed mindset and mathematical exclusion (Rattan, Good, & Dweck, 2012).

We see resilience as important for all learners, not only for those who face significant adversity in their lives. Therefore the next question for us was: what are the important factors in a learning environment that enables learners of mathematics to also learn to be resilient?
The importance of mindset

The growth or incremental mindset is one of the four key aspects of mathematical resilience:

“incremental theorists were more likely than entity theorists to endorse working harder and spending more time studying next time in response to an academic setback, whereas entity theorists were more likely than incremental theorists to report that they would spend less time on the subject and would attempt to avoid it altogether in the future” (Nussbaum & Dweck, 2008, p.600).

The fixed or entity mindset seems to have a detrimental effect on many learners, including those who are placed in 'high ability' groups. Some “high ability pupils” with a fixed mindset may consider that they are better than everyone else, grow complacent and fail to make the progress they could, given the expertise they already have. Others become steadily more anxious that they are “imposters” and will eventually be found out (Boaler, 1997). Neither stance is helpful in encouraging young people to continue to study mathematics.

Having a fixed mindset is also a likely cause of anxiety. If every engagement with mathematics has the potential to expose the actual level of your intellectual abilities which you may worry are low, then every step will cause anxiety and every off-the-cuff remark by the teacher is at risk of being taken as confirmation of low abilities. Where every encounter with mathematics is a threat to the student’s self-esteem and social acceptance (Nussbaum & Dweck, 2008) defence mechanisms for their self-esteem will conflict with mathematical progress. Nussbaum and Dweck (2008) further posit that those students who hold an entity theory are very unlikely to learn from their mistakes because they react to a loss of self-esteem by comparing themselves with others rather than by actively seeking ways which might allow more learning.

Yeager and Dweck (2012) have demonstrated that it is possible to convince learners to abandon a fixed way of thinking about mathematics and adopt a growth theory:
“Our research and that of our colleagues show that if students can be redirected to see intellectual ability as something that can be developed over time with effort, good strategies, and help from others, then they are more resilient when they encounter the rigorous learning opportunities presented to them.” (Yeager & Dweck, 2012, p.306)

Thus a learning environment that enables learners to develop mathematical resilience must help learners to see their intellectual ability as something that can be grown.

Optimism and Resilience
The resilience that is required to learn mathematics relates strongly to the idea of optimism (Seligman 1995) which inclines students to problem solve (Williams 2014); indeed Seligman explained that optimism is a form of resilience. Optimism is seen as a combination of confidence, persistence and perseverance. “Optimism (resilience) is a psychological characteristic possessed by students who are inclined to problem-solve” (Williams 2014 p.407). Mathematical problem-solving is often part of the learning environment that allows for the growth of mathematical resilience.

 Appropriately challenging learning opportunities have to be offered so that learners understand that resilience is needed and that mathematics is not just about finding the right answer quickly. However problem solving can of itself be experienced as a situation of adversity because of the many failures that can occur before success is achieved.

Confidence can be explained as a combination of knowing that you have sufficient resources and technique to succeed in completing a challenge and that you have successfully completed similar work before. Consequently, it is a characteristic that can be grown through successful engagement with mathematical learning. Confidence and persistence, which are part of optimism, are characteristics of successful mathematics learners (Martin 2003), but some confident students are not inclined to problem-solve (Pajares 1996; Dweck 2012; Williams 2013). Thus there can be said to be two types of confidence:
• an optimistic confidence, which allows for perseverance and successful problem solving and the demonstration of resilience and;
• a non-optimistic confidence which is based in an unconnected knowledge of mathematical ideas that does not incline the possessor of this type of confidence to step beyond certain limits.

Learners with an optimistic confidence (Williams 2014) know both how to work at a challenge and how to access the support, techniques and knowledge they need to meet that challenge. They are open to alternative ideas; they see that others’ ideas or suggestions have the potential to extend their own understanding of the situation, affording them an opportunity to see a more efficient way forward and allow their intellectual capabilities to grow even further.

Optimism is used by Seligman (1995) in a non-traditional way, to describe a positive explanatory style where successes are perceived as permanent, the meaning thus differs from the everyday use of the term. Optimism is seen as pervasive, and personal; the optimistic child sees failures as temporary, specific to that given situation, and external (Seligman 1995). The ideas behind optimism help in understanding and explaining resilience (Williams 2014). However in everyday language the term resilience seems to us to convey more meaning to those for whom it was designed to speak, those who are learning or are helping others learn mathematics. Resilience includes confidence, persistence and perseverance. Thus, whilst acknowledging the two terms can be thought of as interchangeable ‘mathematical resilience’ is preferred. Mathematical resilience enables a simple message to be conveyed that public and policy makers will understand, whilst remaining fully grounded in research and maintaining an inner complexity.

The role of persistence and perseverance in resilience

Persistence and perseverance, although often regarded as similar or even the same, can usefully be differentiated when thinking about mathematical learning and resilience. Conroy (1999) demonstrated the difference between persistence and perseverance by drawing attention to
‘adjusting’: “Part of the skill of the power of perseverance is to make those adjustments as you persist” (p. 30). Thom and Pirie (2002) distinguished between the two by describing perseverance as:

...sense (ie. intuitive and experiential) in knowing when to continue with, and not to give up too soon on a chosen strategy or action, and ... knowing when to abandon a particular strategy or action in the search of a more effective or useful one (Thom and Pirie 2002, p.2).

For a mathematically resilient learner it is not sufficient to persist; perseverance is more important. Resilient students report much higher levels of perseverance, intrinsic and instrumental motivation to learn mathematics, mathematics self-efficacy, mathematics self-concept and lower levels of mathematics anxiety than non-resilient students (Lee & Johnston-Wilder 2013).

**Self-efficacy**

Self-efficacy is part of developing mathematical resilience. Bandura defined self-efficacy as “the belief in one’s capabilities to organize and execute the courses of action required to manage prospective situations” (Bandura, 1995, p. 2). Hence the term ‘self-efficacy’ concerns the belief in an individual’s ability to succeed in specific situations. Many of the reasons that adults put forward for not wishing to engage in mathematics are rooted in a belief that they cannot do it; they experience low self-efficacy with regard to mathematics. A strong sense of self-efficacy enables learners to view challenging problems as tasks to be mastered; giving a deep interest in and commitment to the activity and an ability to recover quickly from setbacks (Hoffman, 2010). Sources for self-efficacy are important in our thinking. Bandura (1995) suggests the following four sources:

- A mastery experience: achieving success in situations where there are high levels of demand. Generally over time, with effort and sufficient challenge successfully met, self-efficacy is developed.

- Vicarious experience: observing people demonstrating mastery, if a learner believes they are similar have similar knowledge and skills, then a vicarious experience can prompt the learner to feel more self-efficacious.
- Verbal persuasion: encouragement and positive feedback can contribute to self-efficacy. The encouragement that is important in self-efficacy is a recognition of making appropriate attempts and persevering rather than getting a correct answer.

- Physiological and emotional states influence self-efficacy: someone who regards mathematics with apprehension will not feel self-efficacious and conversely, helping someone overcome their anxieties will help them to become more self-efficacious.

The fourth source of self-efficacy in particular adds to the previous discussions. Anxiety is common in learners when approaching mathematics (Ashcraft 2002) hence we developed the ‘growth zone model’ discussed earlier as a way to discuss the physiological and emotional state of a learner and support learners to position themselves to feel able to maximise learning

**Motivation**

Motivation is also important in learning and mathematical resilience. Motivation, like self-efficacy, is related to belief, in that the amount of motivation a learner has to work towards a particular goal is dependent on how much they believe that they will be able to achieve a particular outcome. Ryan and Deci (2000) argue that when a student has intrinsic motivation, then learning can take place effectively because the individual will seek out ways to fulfil their goals as well as the necessary support. Learners will develop more intrinsic or self-determined motivation to learn, if the learning environment allows for the satisfaction of three innate needs: to feel connected, effective, and agentic when learning. Thus intrinsic motivation can be seen as the result of feeling part of a supportive community and the learner themselves experiencing feelings of agency and control.

Mathematical resilience therefore requires an inclusive environment where all are valued, enabled to execute such actions as will enable them to succeed and where all are able to exercise appropriate control. Thus when learners within an environment are challenged but given choices in ways to complete that challenge and are supported so that their emotional state remains positive
and they succeed with meeting the challenges then that environment can be said to be developing mathematical resilience.

Thus mathematical resilience involves a growth mindset, self-efficacy, motivation and an optimistic confidence. All these factors can be grown in a nurturing and transformative learning environment that is inclusive and helps learners understand the personal value of learning mathematics. Furthermore the literature on motivation and confidence shows that resilient learners must learn how to approach their learning and exercise appropriate choice and control.

**Teaching for Mathematical Resilience**

There is evidence that teaching for mathematical resilience requires system-wide change in western education. Teachers are known to make the biggest difference to the outcomes of any learner (Alton-Lee, 2003), however too many teachers are “teaching to the test” in this era of performativity (Rasmussen, Gustafsson, & Jeffrey, 2014). In every learning environment we have worked in, we have shown that to develop mathematical resilience the learner must feel included in the mathematics community. A focus on developing mathematical resilience means freeing learners from: a fixed mindset which causes avoidance, the anxiety that is in part caused by the testing regime, and the learned helplessness which may be occasioned by path smoothing (Wigley, 1992) and teaching to the test. A focus on developing mathematical resilience can be achieved without adversely affecting results. For example, external test results improved slightly despite troubled times at a school we worked in. The difficulties were occasioned by a high turnover in mathematics teachers and sometimes no mathematics teachers at all (Johnston-Wilder & Lee, 2010b) but a resilient attitude developed in the pupils and some other staff avoided any deterioration in examination results.
Existing research, experiments and experience point the way to how to facilitate the development of mathematical resilience. Learning environments that build mathematical resilience allow learners to have agency, independence and choice so that they can focus on tasks that challenges them. Learners are actively encouraged to work collaboratively, not just work in a group (Swan, 2006), but so that they are active and harness their energies constructively. In such environments, learners can articulate their ideas clearly and safely (Lee, 2006) and see mistakes as part and parcel of optimal learning (Mason et al., 2010). They are encouraged to be curious, to try out ideas, make mistakes and make their own connections (Askew et al., 1997). Learners are likely to experience their mathematical attainment growing. Learners will be encouraged to engage in repetition and consolidation of ideas so that they build familiarity and fluency (Nunes, Bryant, & Watson 2008). However given appropriate tasks, consolidation and repetition can also be part of exploring ideas.

**Coaching for Mathematical Resilience**

Whilst good mathematics teachers are needed if society is to continue to grow its technological base, learning mathematics does not always need the presence of a mathematics teacher, as students need time to work on ideas. Successful outcomes for adult and school age learners have been gained by developing coaches for mathematical resilience who do not purport to “know the answer” but do know how to work resiliently on mathematics (Johnston-Wilder et al., 2013). In this ‘coaching for mathematical resilience’ project, the adult coaches first needed to build their own mathematical resilience before they could begin to support apprentices or adult learners of mathematics, otherwise mathematics anxiety was likely to be ‘transmitted’ (Beilock, Gunderson, Ramirez & and Levine 2010). The idea of journeying together was used as part of the project, along with the metaphor of safely proceeding up a mountain so that the participants could see that it was important that no-one was left behind. It seemed this was a novel idea to the adults; they were used to being allowed to “fall by the wayside” in mathematics. That the others in the group would wait, offer different ways of looking at the ideas and consider each other’s progress to be a matter of conjoint agency was ultimately important in the development of a community of coaches. The
coaches’ mathematical thinking improved with their willingness to engage in reasoning about and doing mathematics. In order to be in their ‘growth’ zone, many had to wrestle against the panic and acute anxiety represented by the ‘danger/red’ zone. However the conjoint agency of the community meant that “coming out of the red” was a joint endeavour; they took care of each other’s emotions.

There is further evidence that parents are well placed to help their children, again not by knowing the answer or what to do mathematically, but by asking questions and helping the learner find the help that they need (Goodall & Johnston-Wilder, 2015); this is discussed in depth in the next chapter.

Conclusions

The construct “mathematical resilience” describes the positive attributes that learners require in order to be prepared to engage with, learn and use mathematics both at school and, perhaps more importantly, beyond. It is needed because of the negative emotions and exclusion which have been seen to be engendered by many traditional approaches to teaching mathematics (Nardi and Stewart 20013, Ashcraft 2002). Mathematical resilience can be developed in learners by the use of specific approaches to teaching and learning but also by offering explicit and targeted support by coaches and parents, who do not know the answer but can encourage resilient learning behaviour in seeking to overcome barriers and find solutions.

Mathematical resilience is based in the emotional attributes that are demonstrated by those who have a positive stance to mathematics. There are four aspects to mathematical resilience: growth, value, knowing that how to work at mathematics and knowing how to gain support in pursuing mathematical learning. Those with mathematical resilience have a growth theory of learning; they know that they can grow their mathematical abilities. Those who are happy to engage with mathematics understand its value and know themselves to be a valued, included member of a mathematical learning community. Importantly those learners with mathematical resilience understand that success in learning and using mathematics will involve a certain amount of struggle.
They know that mathematics does not always offer a smooth path to an answer and often needs perseverance. However resilient learners also know that they are not struggling with mathematics alone, there is a community from which support can be gained.

The construct of mathematical resilience grew out of the wider psychological construct of general resilience. Whilst acknowledging that all learning requires a certain amount of resilience we have encountered many learners who show resilience in diverse aspects of their lives, but seem unable or unwilling to bring that resilience to the learning of mathematics. Hence there is a need to delineate the specific construct of mathematical resilience.

Mathematical resilience enlists ideas from constructs such as mindset (Dweck, 2000), optimism (Seligman, 1995), self-efficacy (Bandura, 1995) and motivation (Ryan and Deci, 2000) in order to present a meaningful but complex construct in itself. We have found that the ideas encapsulated by mathematical resilience speak to learners, teachers, parents and others entrusted with overseeing mathematical learning in society. Importantly the simple message of developing mathematical resilience also seems to be gaining the attention of policy makers. More research will be needed before we know for sure how the complexities of mathematical resilience are played out within the whole mathematics education system.

The ideas captured by mathematical resilience are spreading, as more people concerned about mathematics education come to know about and use the construct. As mathematical resilience is built on established ideas and research, this gives reason to hope that the negativity that surrounds mathematics can be dissipated and replaced with a positive image of agency, empowerment and control.

References


